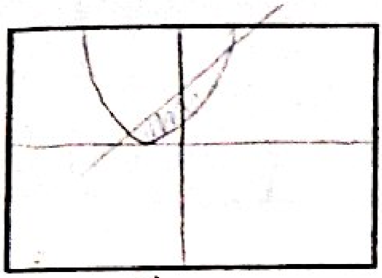
Key

Area of a Region Between Two Curves - Homework

For each problem, sketch the region bounded by the graphs of the functions and find the region of the area.

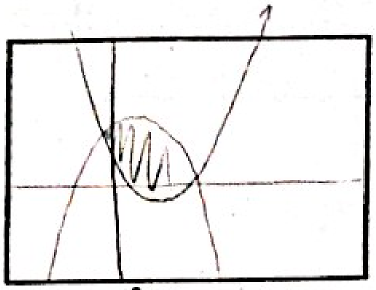
1. $y=x^2+2x+1, y=2x+5$



Integral: $\int_{-2}^{2} (2x+5)-(x^2+2x+1)$

Area: $32/3$

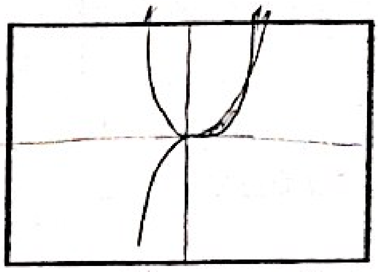
2. $y=x^2-4x+3,$ $y=-x^2+2x+3$



Integral: $\int_0^3 (-x^2+2x+3)-(x^2-4x+3)$

Area: 9

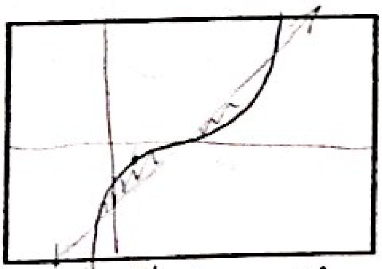
3. $y=x^2$, $y=x^3$



Integral: $\int_0^1 (x^2-x^3)\,dx$

Area: $1/12$

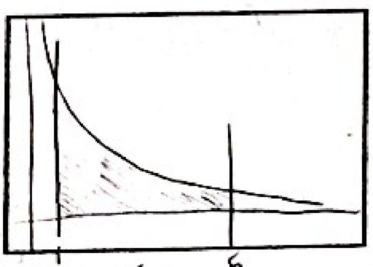
4. $y=(x-1)^3, y=x-1$



Integral: $\int_0^1 (x-1)^3-(x-1) + \int_1^2 (x-1)-(x-1)^3$

Area: $1/2$

5. $y=\frac{1}{x^2}, y=0, x=1, x=5$

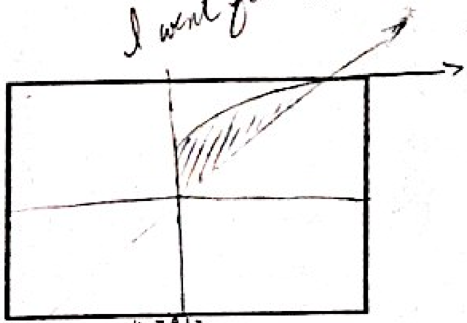


Integral: $\int_1^5 \frac{1}{x^2}\,dx$

Area: $4/5$

6. $y=\sqrt{3x}+1, y=x$

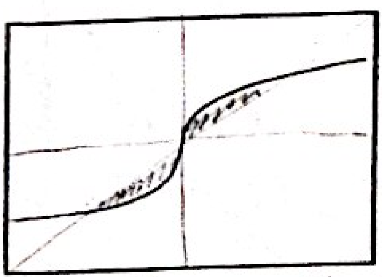
I went from zero



Integral: $\int_0^{4.7913} (\sqrt{3x}+1-x)\,dx$

Area: 5.423

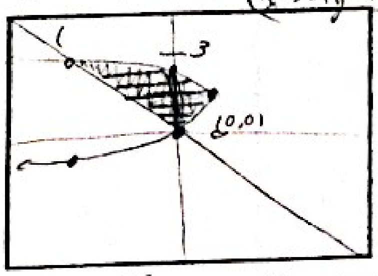
7. $y=\sqrt[3]{x}, y=x$



Integral: $\int_{-1}^0 (x-x^{1/3}) + \int_0^1 (x^{1/3}-x)$

Area: $1/2$

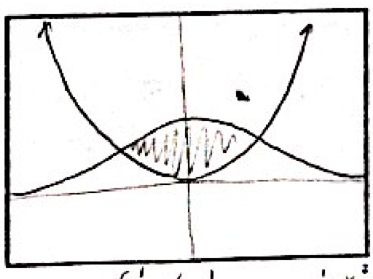
8. $x=2y-y^2, x=-y$



Integral: $\int_0^3 (2y-y^2)-(-y)$

Area: $9/2$

9. $y=\frac{1}{1+x^2}, y=\frac{1}{2}x^2$



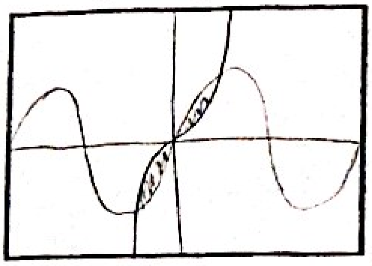
Integral: $\int_{-1}^1 \left(\frac{1}{1+x^2} - \frac{1}{2}x^2\right)$

Area: 1.24 $\frac{\pi}{2}-\frac{1}{3}$

$.4812$

$y = 3x - 2$

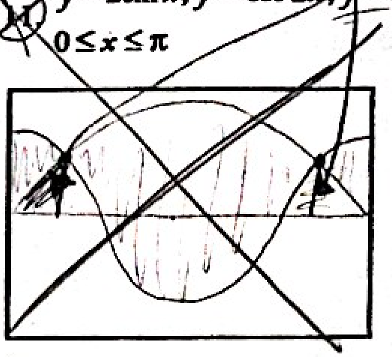
~~10.~~ $y = 2\sin x,\ y = \tan x$
$-\frac{\pi}{3} \le x \le \frac{\pi}{3}$



Integral: $\int_{-\pi/3}^{0} (\tan x - 2\sin x) + \int_{0}^{\pi/3} (2\sin x - \tan x)$

Area: $.614$

~~11.~~ $y = 2\sin x,\ y = \cos 2x,\ y = 0$
$0 \le x \le \pi$

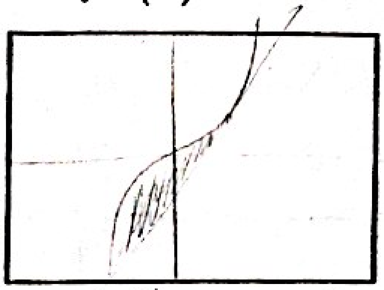


Integral:

Area: 4.808

$\int_{0}^{.3747} (\cos 2x - 2\sin x) + \int_{.3747}^{2.766} (2\sin x - \cos 2x) + \int_{2.766}^{\pi} = .202 + 4.404 + .202$

12. $y = x^3$ and the tangent to y at $(1,1)$



Integral: $\int_{-2}^{1} (x^3 - (3x - 2))$

Area: $27/4$

$y + x^2 = 16$
$x^2 = 16 - y$
$x = \sqrt{16 - y}$

13. Find the value of b that divides the region between $y = 16 - x^2$ and the x and y-axis into two equal areas if the line is

a) vertical (in the form of $x = b$)

$x = 1.39$

$A = \frac{128}{3}$ of whole region

$\int_0^b (16 - x^2)\,dx = \int_b^4 (16 - x^2)\,dx$

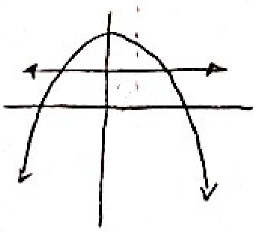
$16x - \frac{x^3}{3}\Big]_0^b = 16x - \frac{x^3}{3}\Big]_b^4$

$16b - \frac{b^3}{3} = 64 - \frac{64}{3} - 16b + \frac{b^3}{3}$

$32b - \frac{2}{3}b^3 = \frac{128}{3}$ Graph

b) horizontal (in the form of $y = b$).

$x = \sqrt{16 - y}$

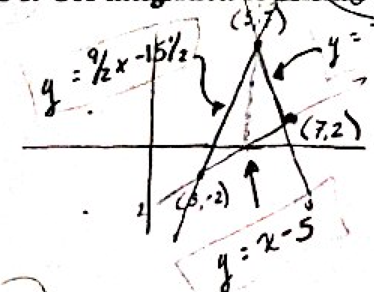


$\int_0^b \sqrt{16 - y}\,dy$ $\quad u = 16 - y$, $du = -dy$

$-\frac{2(16 - y)^{3/2}}{3}\Big]_0^b = \frac{64}{3}$

$b = 5.9206$

14. Use integration to to find the area of the triangle having the vertices (3, -2), (5, 7), and (7, 2)



$y = \frac{9}{2}x - 15\frac{1}{2}$

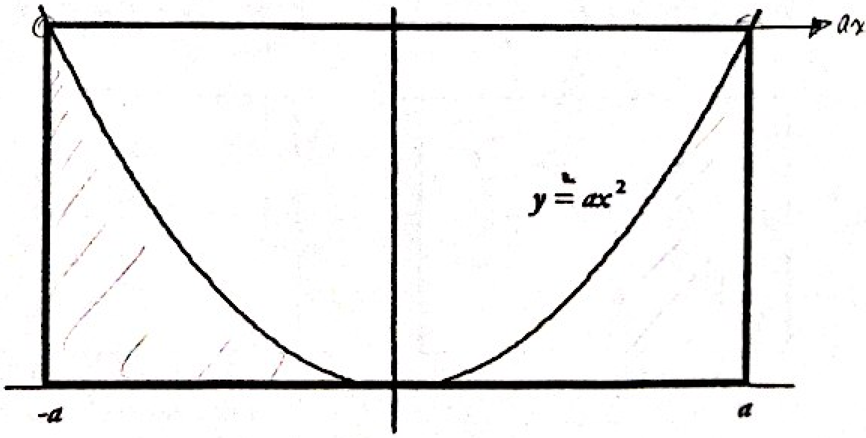
$y = -\frac{5}{2}x + 19.5$

$y = x - 5$

$\int_3^5 \left(\frac{9}{2}x - 15\frac{1}{2}\right) - (x - 5) + \int_5^7 \left(\frac{-5}{2}x + 19\frac{1}{2}\right) - (x - 5)$

$7 + 7 = 14$

15. Show that the area of the function $y = ax^2$ is $\frac{2}{3}$ of the area of the circumscribed rectangle.



$A_{\square} = ax^2 \cdot 2a = 2a^2x^2$ at $x = a$, $2a^4$

$\int_{-a}^{a} ax^2\,dx = \frac{ax^3}{3}\Big]_{-a}^{a} = \frac{a^4}{3} - \left(\frac{-a^4}{3}\right) = \frac{2a^4}{3}$

Area under curve is 1/3 of area of rect, so area above curve is 2/3 of area of rect.